Zolotarev Bandpass Filters

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***Abstract* — Zolotarev rational functions may be used in certain bandpass filter applications for which two narrower passbands are required. Coupled-resonator lowpass prototypes for narrow bandpass filters based on even- and odd-degree Zolotarev functions are synthesized using transformed variables.**

I. INTRODUCTION

A double-passband rf or microwave filter requirement is usually satisfied with two bandpass filters, diplexed at both ends. In instances where rejection between the passbands is not required, a more efficient use of resonators and a simpler structure may be obtained with a single coupled-resonator filter based on a Zolotarev lowpass prototype of either even or odd degree.

The conventional Chebyshev LC lowpass filter has an equiripple passband in the normalized lowpass frequency domain defined by $0 \leq \omega \leq 1$. The Zolotarev lowpass filter is similar, except that the equiripple passband is defined by $0 < a \leq \omega \leq 1$, as for a conventional LC bandpass filter. The even-degree Zolotarev lowpass filter has a mismatch at zero frequency, and is realizable as an impedance-transforming LC ladder network with uneven terminations [1]. The odd-degree Zolotarev lowpass filter has a single reflection zero (loss zero) at zero frequency and a large ripple between zero frequency and the lower passband corner, and is realizable in LC ladder form with equal terminations [2].

Zolotarev rational function approximations for lowpass filters of even- and odd-degree were introduced by Matthaei [1] and Levy [2], respectively, and Horton extended the odd-degree Zolotarev approximation to responses with finite-frequency transmission zeros (loss poles) [3]. Even-degree approximation was achieved by mapping a Chebyshev response, while odd-degree Zolotarev approximation required specialized computational techniques using Jacobi's eta function.

In this paper, classical Chebyshev rational function approximation in a transformed variable is extended to include Zolotarev responses, thus simplifying the design procedure. The resulting amplitude response can be easily converted to that of a narrow bandpass filter and used to realize a coupled-resonator lowpass prototype network.

II. TRANSFORMED VARIABLE SYNTHESIS

The conventional LC bandpass transformed frequency variable is given by [4]

$$z^2 = \frac{\omega^2 - 1}{\omega^2 - a^2}, \ \mathrm{Re}(z) \geq 0, \tag{1}$$

which maps the filter passband onto the entire imaginary z-axis and the stopband into the positive z-axis. The starting point for the approximation step of synthesis is formation of the polynomial

$$E + zF = \prod_{i=1}^{n} (m_i + z), \tag{2}$$

where n is the filter degree, the m_i are the loss poles transformed by (1) and E and F are even polynomials in z. Finite nonzero loss poles transform to identical pairs on the real z-axis, or to two positive-real conjugate pairs for each complex s-plane quadruplet. As a result, $E + zF$ is strictly Hurwitz, and the roots of E and zF are interlaced along imaginary z-axis (the filter passband). For stopband (positive z-axis) loss estimates, a further frequency transformation may be made to $\gamma = \ln z$, and $\gamma_i = \ln m_i$.

A. Even Degree Approximation

The even-degree Zolotarev response can be obtained by a straightforward bilinear mapping of the Chebyshev response, where the lower passband corner at $\omega = 0$ is mapped to $\omega = a$, and the upper passband corner at $\omega = 1$ and $\omega \to \infty$ map to their same values [1], [2]. The same results can also be obtained with the transformed variable using (1) and (2), with all $m_i = 1$ (all loss poles at $\omega \to \infty$ and none at zero frequency), using the classical Chebyshev rational function [4]

$$|K|^2 = \frac{k^2 E^2}{E^2 - z^2 F^2}, \tag{3}$$

where $K = S11/S21$ is the characteristic function, $S11$ and $S21$ are the reflection and transmission coefficients, respectively, and

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$$k = \frac{1}{\sqrt{10^{\frac{RL}{10}} - 1}}, \tag{4}$$

where *RL* is the minimum passband return loss ripple.

The rational function (3) may be used for any general distribution of loss poles, and the frequency transformation (1) guarantees that $|K|^2$ will be equiripple with a maximum value of k^2 in the passband.

In approximating a stopband specification by iteratively adjusting the loss poles, the loss α in decibels may be estimated by [4]

$$\alpha \approx -RL - 6.02 + 4.34 \sum_{i=1}^{n} \ln \coth \left| \frac{\gamma - \gamma_i}{2} \right|, \tag{5}$$

with negligible error for values of *RL* and α which are greater than 15 dB each.

B. Odd Degree Approximation

The method used to form the Chebyshev rational function (3) does not apply to an odd-degree Zolotarev rational function because a loss zero is required at $z = 1/a$ ($\omega = 0$), which is outside the passband. However the approximating function can be formed as the product of two rational functions: one which is equiripple in the passband with $n - 1$ loss zeros, and another which is nearly constant across the passband and contains the loss zero at $z = 1/a$. This procedure is similar to that for the doubly-terminated asymmetric parametric LC bandpass filter [4]; the Zolotarev rational function filter is treated as an asymmetric parametric bandpass filter with no loss poles below the passband and with the real-axis loss zero fixed (rather than a free parameter) at zero frequency.

Using (2) and (4), the Zolotarev rational function, which results after canceling common factors in the two rational functions, is

$$|K|^2 = \frac{k^2 (1/a^2 - z^2) U^2}{E^2 - z^2 F^2}, \tag{6}$$

$$U(z^2) = \frac{cE - z^2 F}{b^2 - z^2}, \tag{7}$$

where $c = b^2 a$; the one positive root of $cE - z^2 F$ at $z^2 = b^2$ is factored out to form the polynomial $U(z^2)$, whose remaining roots are in $z^2 < 0$ (the passband). The positive constant *b* is calculated from (7) by [4]

$$b = \frac{1}{a} \frac{zF}{E}\bigg|_{z=b} = \frac{1}{a} \frac{\prod_{i=1}^{n}(m_i + b) - \prod_{i=1}^{n}(m_i - b)}{\prod_{i=1}^{n}(m_i + b) + \prod_{i=1}^{n}(m_i - b)}. \tag{8}$$

Equation (7) is solved iteratively, starting with $b = 1/a$ until *b* no longer changes value.

Note that $|K|^2 = k^2$ at the passband edges ($z^2 = 0$ and $z^2 \to \infty$). The rational function solution is not exactly equiripple; the passband loss deviates slightly from a true equiripple response with maximum deviation in the vicinity of $z^2 = -b^2$, but for practical cases the deviation is usually so small that it is not perceptible in the loss responses.

In estimating the odd-degree stopband loss, the contribution of the loss zero at $z = 1/a$ is taken into account in (5) by including the negative term

$$-4.34 \ln \coth \left| \frac{\gamma + \ln c}{2} \right| < 0. \tag{9}$$

C. Realization

Following either filter approximation step above, the realization step of synthesis may be performed to obtain a lowpass LC ladder [4] or other appropriate structure.

For bandpass applications, either the even- or the odd-degree Zolotarev response can be remapped to a narrow-bandpass response and realized as a coupled-resonator filter [5]. As in the Chebyshev case, a Zolotarev coupled-resonator filter of even- or odd-degree can be structurally symmetric with equal terminations.

III. Coupled Resonator Prototypes

Following are comparisons of lossless Chebyshev and Zolotarev narrow bandpass prototype filters of degree six and seven. The normalized coupled-resonator lowpass prototype for all examples is shown in Fig. 1; all of the loss poles will be at infinite frequency.

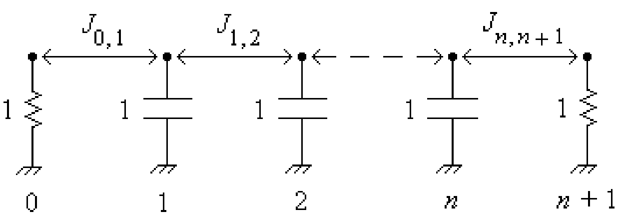

Fig. 1. Normalized coupled-resonator lowpass prototype.

The filter terminations and resonators are represented by unit-valued resistors and capacitors, respectively, and ideal admittance inverters of characteristic admittance $J_{i,i+1}$ represent the couplings.

A. Degree Six Filters

For a six-resonator narrow-bandpass Chebyshev filter centered at 1000 MHz, with an 80 MHz wide equripple passband and 26 dB maximum return loss, theoretical reflection and transmission responses are shown in Fig. 2. The non-zero coupling matrix elements in the prototype, which may be calculated from standard formulas, are

$$J_{0,1} = J_{6,7} = 1.1238$$
$$J_{1,2} = J_{5,6} = 0.9619$$
$$J_{2,3} = J_{4,5} = 0.6564$$
$$J_{3,4} = 0.6184\,.$$

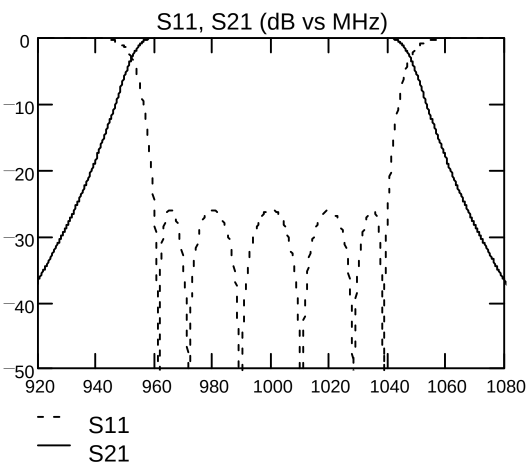

Fig. 2. Lossless Chebyshev bandpass responses, $n = 6$.

Six-resonator Zolotarev responses with the equiripple passbands only in the outer 10 MHz segments of the 80 MHz band are shown in Fig. 3. The corresponding couplings are

$$J_{0,1} = J_{6,7} = 0.6300$$
$$J_{1,2} = J_{5,6} = 0.8984$$
$$J_{2,3} = J_{4,5} = 0.3129$$
$$J_{3,4} = 0.8314\,.$$

Although the Chebyshev filter may be adequate for a typical bandpass filter requirement, it is inefficient when only the outer 10 MHz at each end of the 80 MHz band is actually needed to pass signals, and the Zolotarev filter shows higher stopband rejection. The even-degree Zolotarev filter also provides a small amount of rejection at the center frequency, which could be useful in some applications.

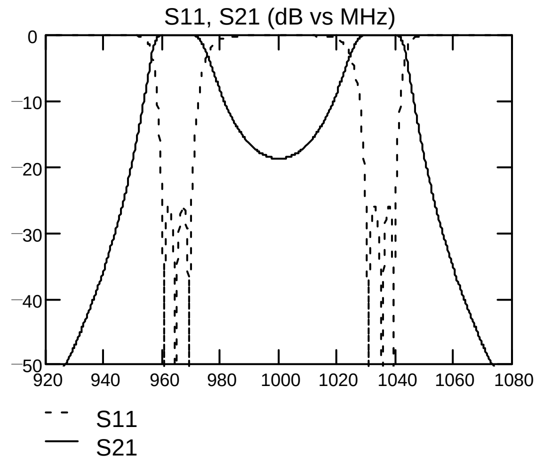

Fig. 3. Lossless Zolotarev bandpass responses, $n = 6$.

B. Degree Seven Filters

Similarly, seven-resonator Chebyshev responses for the same center frequency and equiripple bandwidth are shown in Fig. 4, with couplings given by

$$J_{0,1} = J_{7,8} = 1.1129$$
$$J_{1,2} = J_{6,7} = 0.9417$$
$$J_{2,3} = J_{5,6} = 0.6382$$
$$J_{3,4} = J_{4,5} = 0.5900\,.$$

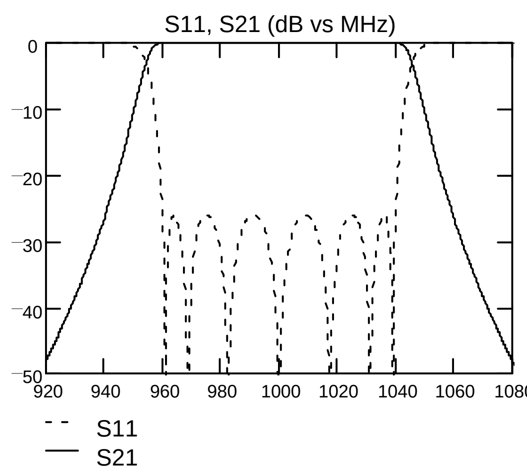

Fig. 4. Lossless Chebyshev bandpass responses, $n = 7$.

Seven-resonator Zolotarev responses, also equiripple only over the outer 10-MHz segments, are shown in Fig. 5. The coupling elements are

$$J_{0,1} = J_{7,8} = 0.6762$$

$$J_{1,2} = J_{6,7} = 0.8533$$

$$J_{2,3} = J_{5,6} = 0.4294$$

$$J_{3,4} = J_{4,5} = 0.6097\ .$$

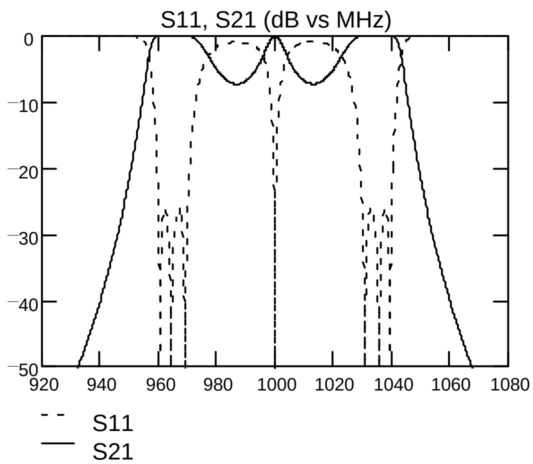

Fig. 5. Lossless Zolotarev bandpass responses, $n = 7$.

Again the Zolotarev filter shows higher stopband rejection than the Chebyshev filter. In the odd-degree Zolotarev filter a narrow passband occurs at center frequency, which could be of use if, for example, a pilot tone must be passed by the filter.

IV. Conclusion

In the examples presented, the location of the loss poles were fixed at infinite frequency in the lowpass frequency domain. Using rational approximation in a transformed frequency variable, the synthesis may include loss poles placed at arbitrary finite stopband frequencies for increased stopband selectivity, or placed on the real *s*-plane axis or at complex *s*-plane frequencies for delay equalization, or both.

In instances where rejection between the passbands is required, but rejection below the lower passband and above the upper passband is not required, the Zolotarev responses may be mapped to a narrow bandstop response [6].

References

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